18.152 PROBLEM SET 4

due April 11th 9:30 am.

You can collaborate with other students when working on problems. However, you should write the solutions using your own words and thought.

Problem 1. *Find a solution to the following problem.*

$$\begin{aligned} &u_{tt} = u_{xx} && 0 < x < 1, t > 0 \\ &u(x,0) = g(x), u_t(x,0) = 0 && 0 \le x \le 1, \\ &u(0,t) = 0, u_x(0,t) = 0 && t \ge 0, \end{aligned}$$

where g is smooth.

Problem 2. *Let Ω be an open bounded smooth domain in $\mathbb{R}^n$. Show that a smooth solution to the following Cauchy-Dirichlet problem is unique.*

$$\begin{aligned} &u_{tt} = \Delta u && x \in \Omega, t > 0 \\ &u(x,0) = g(x), u_t(x,0) = h(x) && x \in \overline{\Omega}, \\ &u(x,t) = f(x) && x \in \partial\Omega, t \ge 0, \end{aligned}$$

where f, g, h are smooth functions.

Hint: Use the energy.

Problem 3. *Solve the following problem, and draw the graph of $u(x, 10)$.*

$$\begin{aligned} &u_{tt} = u_{xx} && x \in \mathbb{R}, t > 0 \\ &u(x,0) = x^2 && x \in \mathbb{R}, \\ &u_t(x,0) = 2x && x \in \mathbb{R}. \end{aligned}$$

Problem 4. *Given smooth functions g, h, solve the following problem.*

$$\begin{aligned} &u_{tt} - u_{tx} - 2u_{xx} = 0 && x \in \mathbb{R}, t > 0 \\ &u(x,0) = g(x) && x \in \mathbb{R}, \\ &u_t(x,0) = h(x) && x \in \mathbb{R}. \end{aligned}$$

Hint: Use the factorization $(\partial_t - 2\partial_x)(\partial_t + \partial_x)$ and modify the d'Alembert formula.